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# *At least* and Quantity Implicature*

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## 1 Introduction

(1) Al hired at least two cooks.

(1) implies speaker ignorance about the number of cooks hired and (2) has an upper-bounding implication that (1) lacks (Horn (1972); Levinson (1983); Krifka (1999)).

(2) Al hired two cooks.

Büring (2008):

- absence of the upper-bounding implication is due to the ignorance implication
- the ignorance implication is a Gricean Quantity Implicature
- the implicature arises because "*at least n* is interpreted as *exactly n or more than n*"

(3) Al hired exactly two cooks or Al hired more than two cooks.

Agenda:

- formulate a classically "neo-Gricean" implementation of Büring's proposal, meeting objections in Coppock & Brochhagen (2013) and Mayr (2013)
- show that such an implementation requires a particular elaboration of the *Standard Recipe* (Sauerland (2004); Fox (2007); Geurts (2011)) for the calculation of Quantity Implicatures
- identify strong Quantity inferences with *at least* that this elaboration accommodates, including "granularity effects" of the sort discussed in Cummins, Sauerland & Solt (2012):

(4) My hometown has at least 10,000 inhabitants.
(adapted from Cummins et al. (2012))

(4) can trigger the inference that the speaker's hometown has fewer than, say, 100,000 inhabitants.

## 2 Background: Neo-Gricean account of Quantity Implicatures

In a classic Neo-Gricean account, (Horn (1972); Fox (2007); Geurts (2011)), upper-bounding implications associated with *scalar items* are drawn on the basis of more informative, formally defined, alternatives to asserted meanings that the speaker could have expressed but chose not to.

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(5) Some of the cooks smoke.

a. Asserted meaning: $\exists$**x[x is a cook & x smokes]**

b. Quantity Implicature: $\neg\forall$**x[x is a cook $\rightarrow$ x smokes]**

(2) Al hired two cooks.

a. Asserted meaning: **[2,...)**

b. Quantity Implicature: $\neg$**[3,...)**

### 2.1 Alternatives: Horn scales and the substitution method

Alternative meanings are generated from "Horn scales", by replacing one or more lexical items in the asserted utterance with a Horn scale mate (Horn (1972), Sauerland (2004)).

(6) Horn scale: {*one*, *two*, *three*, ...}

(2) Al hired two cooks.

(7) a. Asserted meaning: **[2,...)**

b. Alternatives: **[1,...)**, **[2,...)**, **[3,...)**, ...

### 2.2 Inferences: the Standard Recipe

(8) a. S := the speaker; L := the listener

b. $\square$**p** :$\Leftrightarrow$ S's beliefs include/entail **p**

#### 2.2.1 Implicature, Competence, and Ignorance

(9) *Strong alternative (SA)*
An alternative that is semantically stronger than the asserted meaning

(10) *Primary implicature (PI)*
the inference $\neg\square$**q**, for any SA **q**

(11) FIRST INGREDIENT
For any (relevant) SA **q**, L infers the PI $\neg\square$**q**.

(12) a. SA: **[3,...)**, ...

b. PI: $\neg\square$**[3,...)**, ...

(13)

$$\begin{array}{cccccccl} & & & \dots & & \dots & & \\ & & & [5 & & \dots & ) & \\ & & [4 & 5 & & \dots & ) & \\ & [3 & 4 & 5 & & \dots & ) & \text{strong alternatives} \\ \hline [2 & 3 & 4 & 5 & & \dots & ) & \text{asserted meaning} \end{array}$$

Competence and exclusion (Van Rooij & Schulz (2004); Fox (2007); Geurts (2011))

(14) *Competence assumption (CA)*
An assumption of the form $\Box\mathbf{q} \vee \Box\neg\mathbf{q}$

(15) *Secondary implicature (SI)*
the inference $\Box\neg\mathbf{q}$, for any SA $\mathbf{q}$

(16) *Exclusion*
Strengthening of a PI $\neg\Box\mathbf{q}$ to the SI $\Box\neg\mathbf{q}$

(17) CA: $\Box\mathbf{[3,...)} \vee \Box\neg\mathbf{[3,...)}$, ...
SI: $\Box\neg\mathbf{[3,...)}$, ... (equivalent to PI$\wedge$CA)

For L to make a CA about a SA amounts to excluding this SA. From the SI $\Box\neg\mathbf{[3,...)}$, accepting S's belief, L will infer the upper-bounding implication $\neg\mathbf{[3,...)}$.

Ignorance and symmetry (Sauerland (2004); Fox (2007))

Together with a Quality inference, PIs sometimes entail ignorance implications.

(18) a. *Quality inference (QI)*
for an asserted meaning $\mathbf{p}$, L's inference that $\Box\mathbf{p}$

(19) *Ignorance Implication (II)*
an inference of the form $\neg\Box\mathbf{q} \wedge \neg\Box\neg\mathbf{q}$

(20) *Symmetry*
two SAs $\mathbf{q}$ and $\mathbf{r}$ are *symmetric* (relative to a given assertion meaning $\mathbf{p}$) $:\Leftrightarrow$
the asserted meaning $\mathbf{p}$ entails $\mathbf{q} \vee \mathbf{r}$

Under the Standard Recipe, PIs and QI entail IIs just in case there are two symmetric SAs, two SAs that jointly exhaust the options carved out by the asserted meaning.

(21) Bill applied or Carol applied.

(22)
$$\begin{array}{c} \mathbf{b} \wedge \mathbf{c} \\ \mathbf{b} \quad \mathbf{c} \\ \hline \mathbf{b} \vee \mathbf{c} \end{array}$$

(23) Asserted meaning: $\mathbf{b} \vee \mathbf{c}$

(24) QI: $\Box\mathbf{b}\vee\mathbf{c}$

(25) a. SAs: $\mathbf{b}$, $\mathbf{c}$, ... ($\mathbf{b}$ and $\mathbf{c}$ symmetric)

b. PIs: $\neg\Box\mathbf{b}$, $\neg\Box\mathbf{c}$, ...

(26) a. $\Box\mathbf{b}\vee\mathbf{c}$, $\neg\Box\mathbf{b}$ entail $\neg\Box\neg\mathbf{c}$

b. $\Box\mathbf{b}\vee\mathbf{c}$, $\neg\Box\mathbf{c}$ entail $\neg\Box\neg\mathbf{b}$

(27) *Implicature Base (IB)*
the conjunction of the QI with all the PIs

(28) a. IB: $\Box\mathbf{b}\vee\mathbf{c} \wedge \neg\Box\mathbf{b}\wedge \neg\Box\mathbf{c}$

b. IIs: $\neg\Box\mathbf{b}\wedge\neg\Box\neg\mathbf{b}$, $\neg\Box\mathbf{c}\wedge\neg\Box\neg\mathbf{c}$ (entailed by IB)

### 2.2.2 A condition on exclusion (Sauerland (2004))

(21) Bill applied or Carol applied.

An II about $\mathbf{q}$ is inconsistent with the corresponding CA, and hence with the exclusion of $\mathbf{q}$.

(29) a. CA: $\Box\mathbf{b}\vee\Box\neg\mathbf{b}$, $\Box\mathbf{c}\vee\Box\neg\mathbf{c}$ (each inconsistent with IIs)

b. SIs: $\Box\neg\mathbf{b}$, $\Box\neg\mathbf{c}$ (each inconsistent with IIs)

(30) *Weak Excludability (WE)*
a SA $\mathbf{q}$ is *weakly excludable* $:\Leftrightarrow$ $\Box\neg\mathbf{q}$ is consistent with IB.

(31) **SECOND INGREDIENT**
For any SA $\mathbf{q}$ that is not WE, L does not strengthen the PI $\neg\Box\mathbf{q}$ to the SI $\Box\neg\mathbf{q}$.

(21) Bill applied or Carol applied.

(32)
$$\begin{array}{ll}
\mathbf{b}\wedge\mathbf{c} & \text{WE} \\
\mathbf{b}\ \ \mathbf{c} & \cancel{\text{WE}} \\
\hline
\mathbf{b}\vee\mathbf{c} &
\end{array}$$

(2) Al hired two cooks.

(33)
$$\begin{array}{cccccc|l}
 & & & \mathbf{...} & \mathbf{...} & & \text{WE} \\
 & & & [\mathbf{5} & \mathbf{...} & ) & \text{WE} \\
 & & [\mathbf{4} & \mathbf{5} & \mathbf{...} & ) & \text{WE} \\
 & [\mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{...} & ) & \text{WE} \\
\hline
[\mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{...} & ) &
\end{array}$$

(34) For any SA $\mathbf{q}$,

i. there is a SA $\mathbf{r}$ such that $\mathbf{q}$ and $\mathbf{r}$ are symmetric $\Leftrightarrow$ $\mathbf{q}$ gives rise to an II $\Leftrightarrow$ $\mathbf{q}$ is not WE $\Leftrightarrow$ SR prohibits exclusion of $\mathbf{q}$

ii. there is no SA $\mathbf{r}$ such that $\mathbf{q}$ and $\mathbf{r}$ are symmetric $\Leftrightarrow$ $\mathbf{q}$ does not give rise to an II $\Leftrightarrow$ $\mathbf{q}$ is WE $\Leftrightarrow$ SR permits exclusion of $\mathbf{q}$ (subject to CA)

#### 2.2.3 Exclusion by default (Geurts (2011))

Listeners seem to infer upper-bounding implications even in the absence of established information about speaker competence. This suggests that listeners adopt CA by default.

(35) THIRD INGREDIENT
For any SA **q** that is WE, L assumes by default that $\Box\mathbf{q} \vee \Box\neg\mathbf{q}$.

## 3 *At least* under the Standard Recipe

(1) Al hired at least two cooks.

### 3.1 Ignorance from symmetry

Schwarz & Shimoyama (2010) and Mayr (2013) note that IIs with *at least*, and hence the absence of upper-bounding implications, could be derived under SR by positing symmetric SAs.

(36) $$\frac{\mathbf{[2]}\quad \mathbf{[3}\quad \mathbf{4}\quad \mathbf{5}\quad \mathbf{...}\quad )}{\mathbf{[2}\quad \mathbf{3}\quad \mathbf{4}\quad \mathbf{5}\quad \mathbf{...}\quad )}$$

(37) Asserted meaning: **[2,...)**

In the case at hand, then, *at least* is taken to be truth conditionally inert.

(38) a. SAs: **[2]**, **[3,...)** (symmetric)
b. PIs: $\neg\Box$**[2]**, $\neg\Box$**[3,...)**

(39) a. IB: $\Box$**[2,...)** $\wedge\neg\Box$**[2]** $\wedge\neg\Box$**[3,...)**
b. IIs: $\neg\Box$**[2]**$\wedge\neg\Box\neg$**[2]**, $\neg\Box$**[3,...)**$\wedge\neg\Box\neg$**[3,...)** (entailed by IB)

(40) a. CAs: $\Box$**[2]**$\vee\Box\neg$**[2]**, $\Box$**[3,...)**$\vee\Box\neg$**[3,...)** (each inconsistent with IB)
b. SIs: $\Box\neg$**[2]**, $\Box\neg$**[3,...)** (each inconsistent with IB)

(41) $$\frac{\mathbf{[2]}\quad \mathbf{[3}\quad \mathbf{4}\quad \mathbf{5}\quad \mathbf{...}\quad )\quad \cancel{\text{WE}}}{\mathbf{[2}\quad \mathbf{3}\quad \mathbf{4}\quad \mathbf{5}\quad \mathbf{...}\quad )}$$

### 3.2 A scale mate for *at least*

Mayr (2013) suggests a two-scale account for deriving symmetric alternatives under the substitution method.

(42) (i) Horn scale: {*one*, *two*, *three*, ...}
(ii) Horn scale: {*at least*, *exactly*}

(43) SAs: **[2]**, **[3,...)**, ...

A straightforward neo-Gricean explication emerges of Büring's idea: like disjunction, *at least* invokes symmetric SAs derived by substitution.

This addresses Coppock & Brochhagen's (2013) concern that Büring's idea requires *at least* statements to be considered disjunctive in form at some level of syntactic representation.

### 3.3 <u>**Breaking symmetry**</u> (Schwarz & Shimoyama (2010); Mayr (2013))

Fox (2007) observes that IIs with disjunction are obviated under universal operators, and that this is predicted under the Standard Recipe, since universal operators break symmetry.

This correct prediction carries over to *at least*: (44) is consistent with S being fully competent with regard to how many cooks each manager hired.

(44) Every manager hired at least two cooks.

(45) Asserted meaning: $\forall\mathbf{[2,...)}$

(46) a. SAs: $\forall\mathbf{[2]}$, $\forall\mathbf{[3,...)}$ (not symmetric)
b. PIs: $\neg\Box\forall\mathbf{[2]}$, $\neg\Box\forall\mathbf{[3,...)}$

(47) a. IB: $\Box\forall\mathbf{[2,...)} \wedge \neg\Box\forall\mathbf{[2]} \wedge \neg\Box\forall\mathbf{[3,...)}$ (does not entail II)

(48) a. CAs: $\Box\forall\mathbf{[2]} \vee \Box\neg\forall\mathbf{[2]}$, $\Box\forall\mathbf{[3,...)} \vee \Box\neg\forall\mathbf{[3,...)}$ (consistent with IB)
b. SIs: $\Box\neg\forall\mathbf{[2]}$, $\Box\neg\forall\mathbf{[3,...)}$ (consistent with IB)

## 4 Mayr's challenge

(1) Al hired at least two cooks.

The two-scale account derives additional strong alternatives.

(49)

$$
\begin{array}{llllll}
 & & & \ldots & \ldots & \\
 & & \mathbf{[4]} & \mathbf{[5} & \ldots & ) \\
 & \mathbf{[3]} & \mathbf{[4} & \mathbf{5} & \ldots & ) \\
\mathbf{[2]} & \mathbf{[3} & \mathbf{4} & \mathbf{5} & \ldots & ) \;\; \cancel{WE} \\
\hline
\mathbf{[2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \ldots & )
\end{array}
$$

The additional alternatives do not affect IB, as the additional PIs are entailed by $\neg\Box\mathbf{[3,...)}$.

(50) IB: $\Box\mathbf{[2,...)} \wedge \neg\Box\mathbf{[2]} \wedge \neg\Box\mathbf{[3,...)}$

But Mayr (2013) notes that the alternatives **[4]**, **[5]**, etc. are WE. He concludes that under the Standard Recipe, these alternatives jointly derive an upper-bounding implication, viz. $\neg\mathbf{[4,...)}$.

(51)

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | **...** | ... | | |
| | | **[4]** | [5 | ... | ) | |
| | [3] | [4 | 5 | ... | ) | |
| [2] | [3 | 4 | 5 | ... | ) | ~~WE~~ |
| **[2** | **3** | **4** | **5** | **...** | **)** | |

(52) SIs: □¬**[4]**, □¬**[5]**, ... (each consistent with IB)

According to Mayr, this inference is "clearly not attested". If so, there is a flaw either in the two-scale derivation of the alternatives, or in the Standard Recipe. I will focus on the latter option.[1]

# 5 Understanding the challenge

## 5.1 Other possible exclusions

(1) Al hired at least two cooks.

The potential SIs that Mayr (2013) draws attention to are not the only ones that the Standard Recipe permits.

(53)

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | ... | ... | | |
| | | [4] | [5 | ... | ) | |
| | **[3]** | [4 | 5 | ... | ) | |
| [2] | [3 | 4 | 5 | ... | ) | ~~WE~~ |
| **[2** | **3** | **4** | **5** | **...** | **)** | |

(54) SI: □¬**[3]** (consistent with IB)

In fact, *each* of the additional alternatives is WE, so the Standard Recipe permits any one of them to be excluded.

(55)

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | **...** | **...** | | WE |
| | | **[4]** | **[5** | **...** | **)** | WE |
| | **[3]** | **[4** | **5** | **...** | **)** | WE |
| **[2]** | **[3** | **4** | **5** | **...** | **)** | ~~WE~~ |
| **[2** | **3** | **4** | **5** | **...** | **)** | |

(56) SIs: □¬**[3]**, □¬**[4,...)**, □¬**[4]**, ... (each consistent with IB)

1 For the former option, see Schwarz (2013). Mayr (2013) does not pursue either option, but instead attempts an account embedded in the grammatical theory of quantity implicature (Fox (2007); Chierchia, Fox & Spector (2011)).

## 5.2 Strong Excludability

(1) Al hired at least two cooks.

But the set of *all* of these exclusions is not consistent with IB. Instead, there is a family of maximal exclusions consistent with IB.

(57) a.

$$\begin{array}{cccccc|l} & & & \dots & \dots & & \text{WE} \\ & & [4] & [5 & \dots & ) & \text{WE} \\ & [3] & [4 & 5 & \dots & ) & \text{WE} \\ [2] & [3 & 4 & 5 & \dots & ) & \cancel{\text{WE}} \\ \hline [2 & 3 & 4 & 5 & \dots & ) & \end{array}$$

b.

$$\begin{array}{cccccc|l} & & & \dots & \dots & & \text{WE} \\ & & [4] & [5 & \dots & ) & \text{WE} \\ & [3] & [4 & 5 & \dots & ) & \text{WE} \\ [2] & [3 & 4 & 5 & \dots & ) & \cancel{\text{WE}} \\ \hline [2 & 3 & 4 & 5 & \dots & ) & \end{array}$$

... ...

For every SA **q**, there is maximal set of exclusions consistent with IB that *does not* contain $\Box\neg\mathbf{q}$. So no SA is *strongly excludable*:[2]

(58) *Strong Excludability (SE)*
A SA **q** is *strongly excludable* :⇔ $\Box\neg\mathbf{q}$ is an element of every maximal set of SIs whose conjunction is consistent with IB.

(59)

$$\begin{array}{cccccc|ll} & & & \dots & \dots & & \text{WE} & \cancel{\text{SE}} \\ & & [4] & [5 & \dots & ) & \text{WE} & \cancel{\text{SE}} \\ & [3] & [4 & 5 & \dots & ) & \text{WE} & \cancel{\text{SE}} \\ [2] & [3 & 4 & 5 & \dots & ) & \cancel{\text{WE}} & \cancel{\text{SE}} \\ \hline [2 & 3 & 4 & 5 & \dots & ) & & \end{array}$$

In the other examples discussed above, the SE and WE properties coincide.

(21) Bill applied or Carol applied.

(60)

$$\begin{array}{c|l} \mathbf{b\wedge c} & \text{WE} \\ b \quad c & \cancel{\text{WE}} \\ \hline \mathbf{b\vee c} & \end{array}$$

Here the only maximal set of exclusions consistent with IB is $\{\Box\neg\mathbf{b\wedge c}\}$. Hence only $\mathbf{b\wedge c}$ is SE:

2 "Strong excludability" is a neo-Gricean adaptation of Fox's (2007) notion of "innocent exclusion".

(61)
$$\begin{array}{l l}
\mathbf{b{\wedge}c} & \text{WE}\ \ \text{SE} \\
\mathbf{b\ \ c} & \cancel{\text{WE}}\ \ \cancel{\text{SE}} \\
\hline
\mathbf{b{\vee}c} &
\end{array}$$

(2) Al hired two cooks.

(33)
$$\begin{array}{c c c c c c l}
 & & & \mathbf{...} & \mathbf{...} & & \text{WE} \\
 & & & \mathbf{[5} & \mathbf{...} & ) & \text{WE} \\
 & & \mathbf{[4} & \mathbf{5} & \mathbf{...} & ) & \text{WE} \\
 & \mathbf{[3} & \mathbf{4} & \mathbf{5} & \mathbf{...} & ) & \text{WE} \\
\hline
\mathbf{[2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{...} & ) &
\end{array}$$

Here the only maximal set of exclusions consistent with IB is the set of all possible SIs. Hence every SA is SE.

(62)
$$\begin{array}{c c c c c c l}
 & & & \mathbf{...} & \mathbf{...} & & \text{WE}\ \ \text{SE} \\
 & & & \mathbf{[5} & \mathbf{...} & ) & \text{WE}\ \ \text{SE} \\
 & & \mathbf{[4} & \mathbf{5} & \mathbf{...} & ) & \text{WE}\ \ \text{SE} \\
 & \mathbf{[3} & \mathbf{4} & \mathbf{5} & \mathbf{...} & ) & \text{WE}\ \ \text{SE} \\
\hline
\mathbf{[2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{...} & ) &
\end{array}$$

### 5.3 Two targets for revision

(11) **FIRST INGREDIENT**
For any (relevant) SA **q**, L infers the PI $\neg\Box\mathbf{q}$

(31) **SECOND INGREDIENT**
For any SA **q** that is not WE, L does not strengthen the PI $\neg\Box\mathbf{q}$ to the SI $\Box\neg\mathbf{q}$.

(35) **THIRD INGREDIENT**
For any SA **q** that is WE, L assumes by default that $\Box\mathbf{q}\vee\Box\neg\mathbf{q}$.

The SECOND INGREDIENT and the THIRD INGREDIENT refer to the WE property, so each of these ingredients is a conceivable target for revision, replacing WE with SE.

## 6 A strong condition on exclusion?

(1) Al hired at least two cooks.

(63) **REVISED SECOND INGREDIENT**
For any SA **q** that is not SE, L does not strengthen the PI $\neg\Box\mathbf{q}$ to the SI $\Box\neg\mathbf{q}$.

(59)
$$\begin{array}{cccccc l}
 & & & \textbf{...} & \textbf{...} & & \text{WE } \cancel{\text{SE}} \\
 & & \textbf{[4]} & \textbf{[5} & \textbf{...} & ) & \text{WE } \cancel{\text{SE}} \\
 & \textbf{[3]} & \textbf{[4} & \textbf{5} & \textbf{...} & ) & \text{WE } \cancel{\text{SE}} \\
\textbf{[2]} & \textbf{[3} & \textbf{4} & \textbf{5} & \textbf{...} & ) & \cancel{\text{WE}} \ \cancel{\text{SE}} \\
\hline
\textbf{[2} & \textbf{3} & \textbf{4} & \textbf{5} & \textbf{...} & ) &
\end{array}$$

This version obliges L to refrain from exclusion of an SA **q** that is not SE even if **q** is WE, i.e. excluding **q** would not incur any inconsistency. By the same token, it obliges L to not adopt CAs that would guarantee such exclusions, even in the face of compelling evidence for S's competence.

## 7 Mayr's challenge qualified

(64) **Scenario**: Al is the manager of a local restaurant chain. Once a month, Al hires cooks. The owner of the chain, S, to a certain extent gives Al free hand in running these hires. But one issue that S invariably keeps herself informed about is whether or not Al hired ***more than three*** cooks in the previous month. Al therefore follows the following established procedure: at the end of each month, Al send S a message, consisting only of the subject line "1" if Al hired ***more than three*** cooks that month, and "0" otherwise. S is currently working with a consultant, L, who is aware of this established procedure.

(65) L: "Do you know whether Al hired any cooks last month?"
S: "Al hired at least two cooks."

The scenario attributes to L the CA $\Box\mathbf{[4,...)}\vee\Box\neg\mathbf{[4,...)}$. S's reply is not judged to lead L to revoke this assumption, contrary to what the REVISED SECOND INGREDIENT demands. Instead, as expected under the original SECOND INGREDIENT, S's reply is judged to likely lead L to draw the inference $\Box\neg\mathbf{[4,...)}$, the upper-bounding SI entailed by the PI $\neg\Box\mathbf{[4,...)}$ and the CA $\Box\mathbf{[4,...)}\vee\Box\neg\mathbf{[4,...)}$.

It seems that Mayr's (2013) challenge only applies in absence of a relevant CA established in context. The SECOND INGREDIENT is then not the right target for revision.

(31) **SECOND INGREDIENT**
For any SA **q** that is not <u>WE</u>, L does not strengthen the PI $\neg\Box\mathbf{q}$ to the SI $\Box\neg\mathbf{q}$.

The original SECOND INGREDIENT correctly allows for contextually supported SIs with *at least* to not be limited to upper-bounding implications.

(66) **Scenario**: Al is the manager of a local restaurant chain. Once a month, Al hires cooks. The owner of the chain, S, to a certain extent gives Al free hand in running these hires. But one issue that S invariably keeps herself informed about is whether or not Al hired ***exactly three*** cooks in the previous month. Al therefore follows the following established procedure: at the end of each month, Al send S a message, consisting only of the subject line "1" if Al hired ***exactly three*** cooks that month, and "0" otherwise. S is currently working with a consultant, L, who is aware of this established procedure.

(65) L: "Do you know whether Al hired any cooks last month?"
S: "Al hired at least two cooks."

S's reply is judged to lead L to likely draw the inference $\Box\neg$**[3]**, the SI entailed by the PI $\neg\Box$**[3]** and the CA $\Box$**[3]**$\vee\Box\neg$**[3]**.

Also, SIs with *at least* do not always require that the relevant CA be given explicitly. Sometimes L might infer a CA by reasoning about the evidential grounds for S's statement.

(4) My hometown has at least 10,000 inhabitants.
(adapted from Cummins et al. (2012))

(4) semantically entails that S's hometown does not have fewer than 10,000 inhabitants. It is reasonable for L to assume that S's evidence for the truth of this asserted content will also settle the question whether S's hometown has 100,000 or more inhabitants.

## 8 A strong condition on competence by default

(1) Al hired at least two cooks.

What remains of Mayr's challenge is the question why SIs with *at least*, in contrast to SIs in other cases, are dependent on the corresponding CAs being established in context.

The answer that suggests itself is that L's default assumptions about S's competence are limited to SAs that are SE.

(67) **REVISED THIRD INGREDIENT**
For any SA **q** that is <u>SE</u>, L assumes by default that $\Box$**q**$\vee\Box\neg$**q**.

## 9 Conclusion

Inferences associated with numerals modified by *at least* are amenable to a straightforward Neo-Gricean analysis. Both the numeral and *at least* are subject to substitution by Horn scale mates. Ignorance implications are credited to symmetry. Secondary implicatures are predicted that upon closer inspection are indeed attested. But they are subject to conditions that suggest a particular elaboration of the proposal that listeners consider speakers competent by default.

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